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ON SYMMETRY OF ELEMENTARY PARTICLES

Abstract

The new quantum number σ is introduced. It is shown that the conservation of σ -number results in the conservation of difference between baryon and lepton numbers obtained earlier by Georgi and Glashow. The conservation of σ -number also predicts that electron type neutrino mass is exactly zero. The quark-lepton symmetry is discussed. It is shown that the nature of quark-lepton symmetry is reflected by the fact that elementary particles of the same generation are subject to the symmetry transformation represented by 4-group of diedr. It is also shown that colorless elementary particles are subject to the same symmetry transformation. The new elementary particles (transbaryons) are predicted.

1 The law of conservation of σ -number.

It is known that upper and lower quarks are positioned asymmetrically on the "charge axis". With the aim to bring in some symmetry to this asymmetric

disposition we will introduce the new additive quantum number σ determined so as to result, in combination with quark electric charge q , the charge of the respective lepton.

Then for u and d quarks we will have, respectively, $\sigma_u = 1/3$ and $\sigma_d = -2/3$. In general, σ -numbers for all quarks and anti-quarks are determined by the following formula:[1]

$$\sigma = q - 1/3, \quad \tilde{\sigma} = \tilde{q} + 1/3. \quad (1)$$

Now the quarks (and leptons) are positioned symmetrically with respect to $\sigma = -q$ axis on (q, σ) plane (see Fig.1). This brings about an idea that together with the law of conservation of electric charge there is realized the second law of conservation, the law of conservation of σ -number. As we will see later, this idea results in extremely valuable findings. In particular, the conservation of σ -number results in conservation of the difference between baryon and lepton numbers obtained earlier by Georgi and Glashow from $SU(5)$ symmetry.[2]

Let us first show that σ -numbers for baryons, mesons, leptons and photons are determined by the following expressions:

$$\sigma_B = Q_B - 1, \quad \sigma_M = Q_M, \quad \sigma_L = Q_L + 1, \quad \sigma_\gamma = 0, \quad \tilde{\sigma} = -\sigma \quad (2)$$

where Q is an electric charge of a particle. Indeed, formula (2) is obvious for baryons and mesons due to their quark structure. Similarly, σ -numbers can be obtained for exotic adrons. Using (2), we will obtain, in particular, $\sigma(p) = 0$, $\sigma(n^0) = -1$, $\sigma(\pi^0) = 0$, $\sigma(\tilde{\Lambda}^0) = 1$, $\sigma(\Sigma^-) = -2$ and so on. Taking into account that $\sigma(\pi^0) = 0$, we will obtain $\sigma_\gamma = 0$ from $\pi^0 \Rightarrow 2\gamma$ decay.

In order to find σ_L , we will give the same quantum number (by analogy with the electric charge) to all charged leptons ($\sigma_e = \sigma_\mu = \sigma_\tau$) and another quantum number to neutral leptons ($\sigma_{\nu_e} = \sigma_{\nu_\mu} = \sigma_{\nu_\tau}$). These numbers are related to each other by the formula $\sigma_{\nu_e} = 1 + \sigma_e$ that can be derived from $n^0 \Rightarrow p + e^- + \tilde{\nu}$ decay. Then from $uu \Leftarrow x \Rightarrow e^+ \tilde{d}$ reaction where the same boson may decay into antilepton + antiquark or quark pair [3] we derive $\sigma_{e^+} = -\sigma_{e^-} = 0$, and $\sigma_\nu = 1$. As we can see, these results are in accordance with expressions (2). After we have derived formula (2), we can check out that the conservation of σ -number takes place for all the reactions observed so far [4]. It does not prohibit also possible proton decay

through channels $p \Rightarrow e^+\pi^0; e^+\pi^+\pi^-$ where conservation of baryon and lepton numbers is violated.

The general formula for conservation of σ -number can be written as follows:

$$\sum Q_B - N_B + \sum \tilde{Q}_B + \tilde{N}_B + \sum Q_M + \sum \tilde{Q}_M + \sum Q_L + N_L + \sum \tilde{Q}_L - \tilde{N}_L = \text{const} \quad (3)$$

where N is the number of particles. Here, taking into account the conservation of electric charge, we will finally obtain the desired result:

$$(N_B - \tilde{N}_B) - (N_L - \tilde{N}_L) = \text{const} \quad (4)$$

It should be pointed out that the baryon number in formula (4) could be replaced by total number of quarks N_k comprised of baryon and meson quark numbers. Indeed, taking into account that for baryons and mesons $(N_k - \tilde{N}_k) = 3(N_B - \tilde{N}_B)$ and $(N_k - \tilde{N}_k) = 0$ and using formula (4), we will obtain the following relation between quarks and leptons

$$(N_k - \tilde{N}_k) - 3(N_L - \tilde{N}_L) = \text{const} \quad (5)$$

As we will see later, the law of conservation of σ -number predicts that electron type neutrino mass is exactly zero. It should be mentioned in this relation that two contradictory theories trying to resolve the problem of neutrino mass have been developed so far by Dirak-Weyl in 1929 and Majoran in 1936. According to the former [5] electron neutrino mass is exactly zero and the conservation of lepton number takes place. According to the latter [6, 7] the electron neutrino mass is not zero and therefore the conservation of lepton number does not take place. The Majoran theory allows neutrinoless double beta decay where two d -quarks decay through channel $d \Rightarrow u + e^-$ with violation of lepton number.

The contemporary theories basing on $SU(5)$ and $SO(10)$ symmetry result in the same contradictory predictions [3] "The $SO(10)$ symmetry allows occurrence of some phenomena prohibited by $SU(5)$ symmetry. In particular, $SU(5)$ theory predicts conservation of difference $B - L$ while conservation of baryon number B and lepton number L does not take place. In $SO(10)$ theory, the difference $B - L$ may not hold constant if sufficient number of

Higgs fields are involved". Approximately the same results are mentioned in the Gell-Mann report.[8]

Thus, the main question relating to neutrino mass remains open. There have been a number of attempts for the past 60 years to find theoretical or experimental solution but the problem still exists.[9, 10] The recent experiments in many scientific centers worldwide have been aimed at observing of neutrinoless double beta decay and neutrino oscillations. Currently several new projects are proposed and huge scientific potential and funds mobilized to get a breakthrough on this fundamental problem. Twenty-three scientific centers are involved in the project MINOS (USA).[11] The underground experiment is aimed at neutrino beams detecting after passing some 730km from Fermi-lab (Viskonsin) to Soudan (Minnesota). It is expected that some of the detected neutrinos will have changed their flavor due to neutrino oscillations. In Japanese project KEK neutrino beam will pass some 230km from an accelerator to the detector Super-Kamiokande. [11] The new Heidelberg project GENIUS is aimed at increasing the sensitivity of Majorana neutrino mass from the present level of at best 0.1ev down to 0.01 or even 0.001ev. [9]

Summarizing, we can say that the newly proposed law of conservation of σ -number prohibits neutrinoless double beta decay and, in compliance with Dirac-Weyl and $SU(5)$ theories, predicts that electron type neutrino mass is exactly zero.

2 Quark-lepton symmetry.

Now we will focus on the problem of quark-lepton symmetry.[3, 5] The essence of the problem is to find such a symmetry among the elementary particles, which would explain why each quark-lepton generation is necessarily comprised of the pair of leptons and pair of quarks. Let us shift to the new system of coordinates (q', σ') with the point of origin in the center of a rectangle $e\nu_e u d$ and axes directed along the axes of symmetry (see Fig1.). The relation with an original system of coordinates is given by the following expressions:

$$q' = \frac{1}{\sqrt{2}}q + \frac{1}{\sqrt{2}}\sigma, \quad \sigma' = -\frac{1}{\sqrt{2}}q + \frac{1}{\sqrt{2}}\sigma - \frac{1}{3\sqrt{2}} \quad (6)$$

The coordinates of the vertices are now as follows:

$$e\left(-\frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{3}\right), \nu_e\left(\frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{3}\right), u\left(\frac{1}{\sqrt{2}}, -\frac{\sqrt{2}}{3}\right), d\left(-\frac{1}{\sqrt{2}}, -\frac{\sqrt{2}}{3}\right); \quad (7)$$

The group that represents the symmetry transformation for the above mentioned rectangle, consists of the following elements: E -identity transformation; turn around axis z for angle 2π ; A -turn around axis σ' for angle π ; B -turn around axis q' for angle π ; C -turn around axis z for angle π .

For these elements we will obtain the group multiplication table (see Table 1).

	E	A	B	C
E	E	A	B	C
A	A	E	C	B
B	B	C	E	A
C	C	B	A	E

Table 1

Under multiplication we understand the subsequent execution of the corresponding operations. The elements of the group have order 2 (except for identity element), since $\chi^2 = E$ and $\chi^{-1} = \chi$, where χ is an arbitrary element of the group. Thus, the totality of the elements E, A, B, C makes up the Abelian group. The "self-transformation" of a regular polygon is expressed by means of the following matrices [12]

$$D_k = \begin{pmatrix} \cos \frac{2\pi k}{n}, & \sin \frac{2\pi k}{n} \\ -\sin \frac{2\pi k}{n}, & \cos \frac{2\pi k}{n} \end{pmatrix}, \quad U_k = \begin{pmatrix} -\cos \frac{2\pi k}{n}, & \sin \frac{2\pi k}{n} \\ \sin \frac{2\pi k}{n}, & \cos \frac{2\pi k}{n} \end{pmatrix}. \quad (8)$$

where $k = 0, 1, 2, \dots, n-1$. These $2n$ -dimensional matrices make up the group of order $2n$ known as the group of diedr. In case of $n=2$ we have the simplest case of the group with elements

$$E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (9)$$

Taking into account that the group (9) is isomorphic to the above mentioned group, we come to conclusion that (9) is the matrix representation of the group. Thus, if particles of the same generation are located in the vertices of rectangle (7) on the plane (q', σ') , this distribution is subject to the symmetry transformation described by the group (9). It is also easy to show that if distribution of particles on the plane (q', σ') is subject to the symmetry transformation described by the group (9) and if coordinates of any arbitrary particle from (7) coincide with one of the vertices, there should be three more particles (and only three, without taking into account the color of the quarks) whose coordinates coincide with the remaining vertices.

Summarizing this paragraph, we can say that the nature of the quark-lepton symmetry could be described by $q\sigma$ -symmetry. It is reflected by the fact that particles of the same generation are subject to the symmetry transformation represented by 4-group of diedr.[13]

3 Symmetry among colorless elementary particles.

It is natural to expect that there is some symmetry among colorless particles as well. We will consider all the baryons and mesons which can be comprised of eighteen known quarks as well as the leptons. Each point on the plane (q, σ) is assumed to "contain" the whole family of the particles (see fig.2). For example, point $(-1; 0)$ contains, besides electron, other charged leptons μ^- and τ^- . It is clear from the picture that there is symmetry with respect to σ' axis. The number of particles in symmetrical points is identical. It is easy to show that the same symmetry exists for multi-quark baryons as well. For example, if baryons with quantum numbers $q = 3, \sigma = -2, B = 5$ consist of eight upper and seven lower quarks, then symmetrical baryons with quantum numbers $q = 2, \sigma = -3, B = 5$ consist of seven upper and eight lower quarks. It is obvious that the number of colorless combinations is identical.

For baryons the same symmetry exists also with respect to other axis q' which is determined by equation $\sigma = q - 3$. It can be proved by direct count of the particles in corresponding points (see fig.3). The dashed lines parallel to q' axis are determined by the following formula:

$$\sigma = q - (B - L) \quad (10)$$

It should be pointed out that formula (2) can be represented by formula (10). For baryons, instead of (10), we have $\sigma = q - B$. The point $q = 3, \sigma = -3, B = 6$ contains only one baryon comprised of all the eighteen known quarks. The distribution of colorless particles shows that, in order to have completed the symmetry, it is necessary to assume some particles not observed so far in the points marked by crosses. For example, there should be three particles in the point with coordinates $q = 4, \sigma = -3$ because there are three particles (leptons) in the symmetrical point. Now the distribution of colorless particles will be subject to the symmetry transformation described by 4-group of diedr (9). The predicted particles should be located in five points symmetrically positioned to those of leptons and mesons. These particles, like leptons, should not have quark structure because for them $B = 6; 7$. They are supposedly ten times heavier than baryons. We will call them transbaryons. The number of transbaryons is less by one of the total number of leptons and mesons. If, by any selection rule, it is prohibited for

some particles to be located in some arbitrary points, then, in compliance with above mentioned symmetry, the same number of particles should be missing out in symmetrically positioned points. But this does not pertain to transbaryons because symmetrical to them leptons and mesons (most of them) have already been observed. The newly proposed symmetry for colorless particles does not depend on generation. Based on this symmetry alone, we cannot judge about forth generation.

Concluding this paragraph, we can say that within the frame of the proposed symmetry for colorless elementary particles the existence of transbaryons seems undisputable .[14]

References

- [1] N.A. Korkhmazyan, Docl. NAN Armenii, Vol. 99 (1999) 182.
- [2] H.Georgi, Sh.Glashow, Phys. Rev. Lett., Vol. 32 (1974) 438.
- [3] L.B.Okun, Physics of elementary particles, (Moscow, 1988).
- [4] European Physical Journal, C, Review of Particle Physics, vol. 3, 1-4, n. 1-794, 1998.
- [5] F.Halzen, A.Martin, Quarks and leptons, (New York, Chichester, Brisbane, Toronto, Singapore, 1984).
- [6] H.Umedzava, Quantum field theory [in Russian], Izd-vo Inostr. Lit-ry, Moscow, 1958.
- [7] Cern Curier, Inter. Jour. Of high energy physics, vol. 38, n. 6, 1998.
- [8] M.Gell-Mann, UFN, vol. 151, n. 4, p. 638, 1987.
- [9] H.V. Klapdor-Kleingrothaus, IAF, vol.61, n. 6, p. 967, 1998.
- [10] S.S.Gershtein, E.P.Martemianov, K.N.Mukhin, UFN, vol. 167, n. 8, p. 811, 1997.
- [11] Fermi News, Fermi National Accelerator Laboratory, vol. 21, n.15, 1997.
- [12] E.Wigner, Group Theory, (New York and London, 1959).

- [13] N.A. Korkhmazyan, N.N. Korkhmazyan, <http://arxiv.org/abs/hep-ph/9912314>, user-Id: hep-ph/9912314, pass: 99jmr
- [14] N.A. Korkhmazyan, N.N. Korkhmazyan, Docl. NAN Armenii (in publ.).

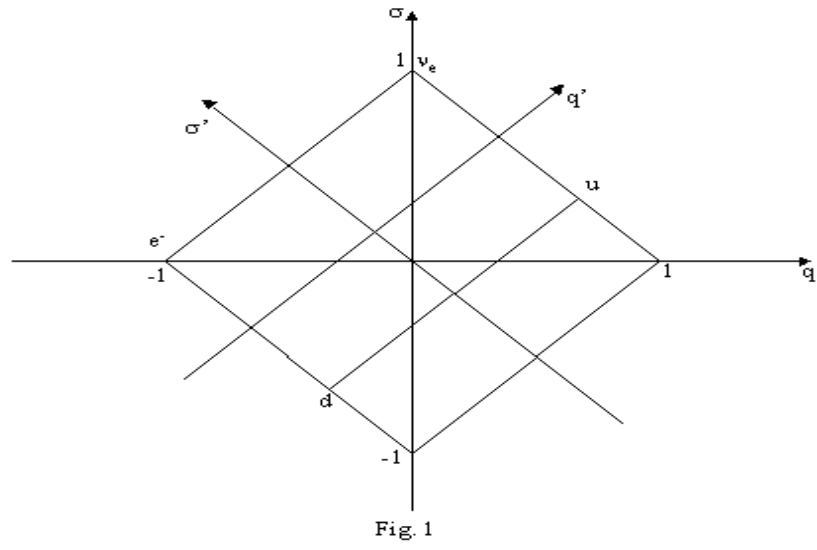


Fig 1. Distribution of quarks and leptons of the first generation on q, σ plane.

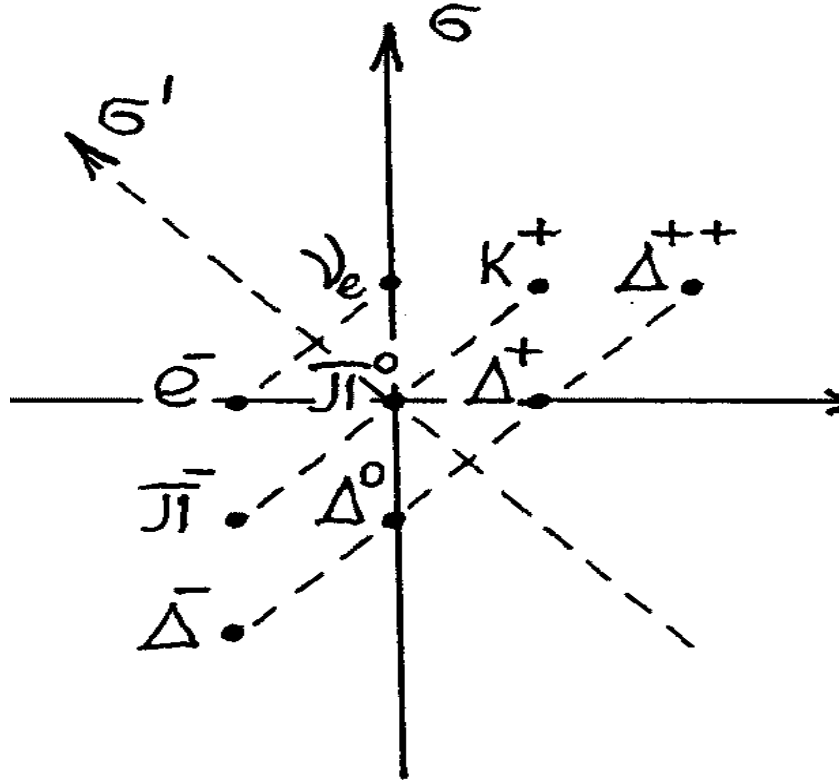


Fig 2. Distribution of ordinary particles on (q, σ) plane. The distance between neighbouring points along axes q and σ is taken as a measurement unit.

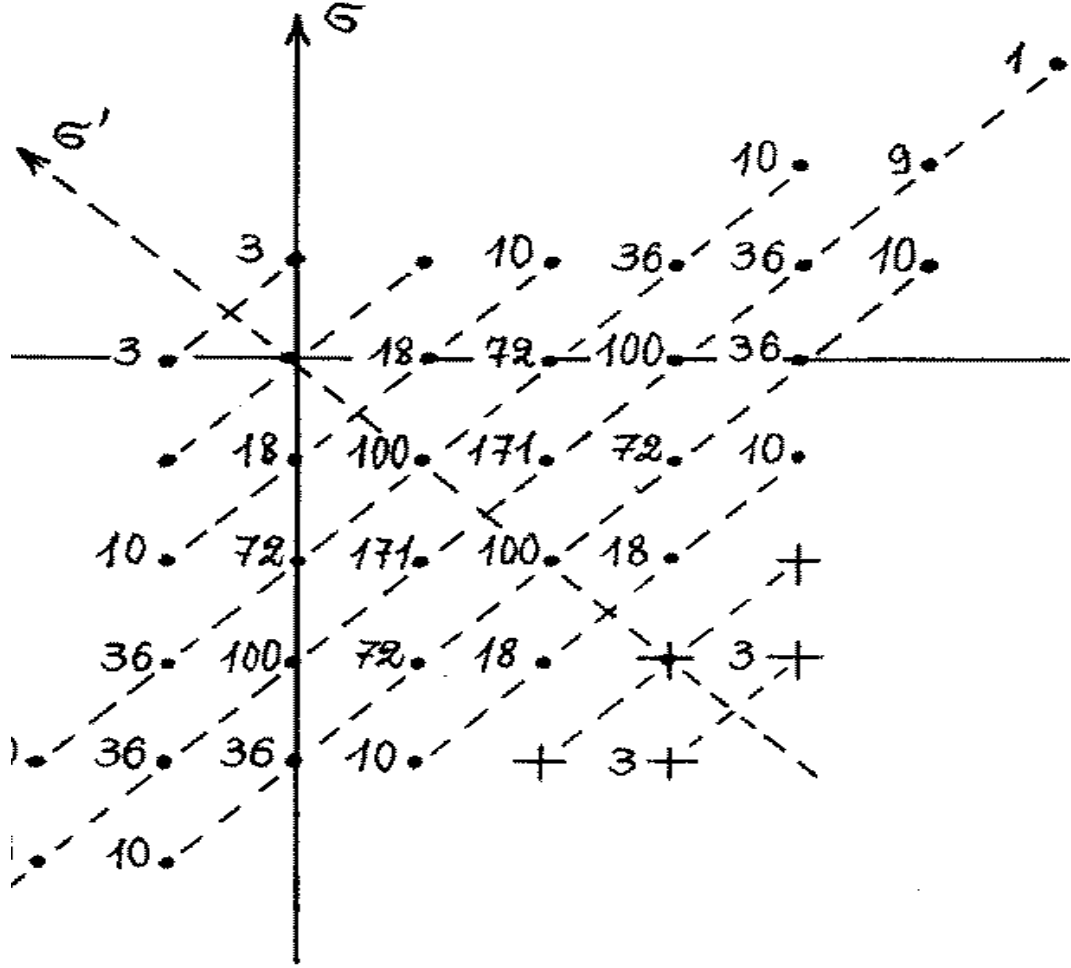


Fig 3. Distribution of colorless particles on (q, σ) plane. Transbaryons are marked by crosses. The figures indicate the number of particles in each point.